OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows part of the curve $y = e^{2x}\cos x$.

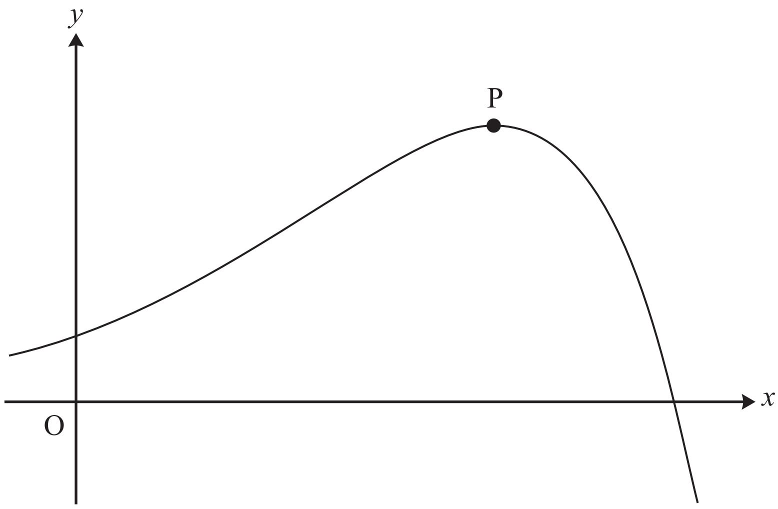

Fig. 1

Find the coordinates of the turning point P. **[6]**

2 Find $\int \sqrt[3]{2x-1}\,dx$. **[4]**

3 Find the exact value of $\int_1^2 x^3 \ln x\, dx$. **[5]**

4 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of $5\,\text{cm}^3$ per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is $V\,\text{cm}^3$.

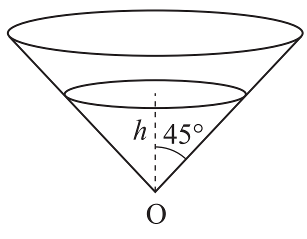

Fig. 4

Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$]. **[5]**

5 A curve has implicit equation $y^2 + 2x\ln y = x^2$.

Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point. **[6]**

6 Solve each of the following equations, giving your answers in exact form.

(i) $6\arcsin x - \pi = 0$. **[2]**

(ii) $\arcsin x = \arccos x$. **[2]**

7 **(i)** The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = \frac{1-x}{1+x},\ x \neq -1.$$

Show that $\mathrm{f}\big(\mathrm{f}(x)\big) = x$.

Hence write down $\mathrm{f}^{-1}(x)$. **[3]**

(ii) The function $\mathrm{g}(x)$ is defined for all real x by

$$\mathrm{g}(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $\mathrm{g}(x)$ is even. Interpret this result in terms of the graph of $y = \mathrm{g}(x)$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the line $y = 1$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{(x-2)^2}{x}$. The curve touches the x-axis at P(2, 0) and has another turning point at the point Q.

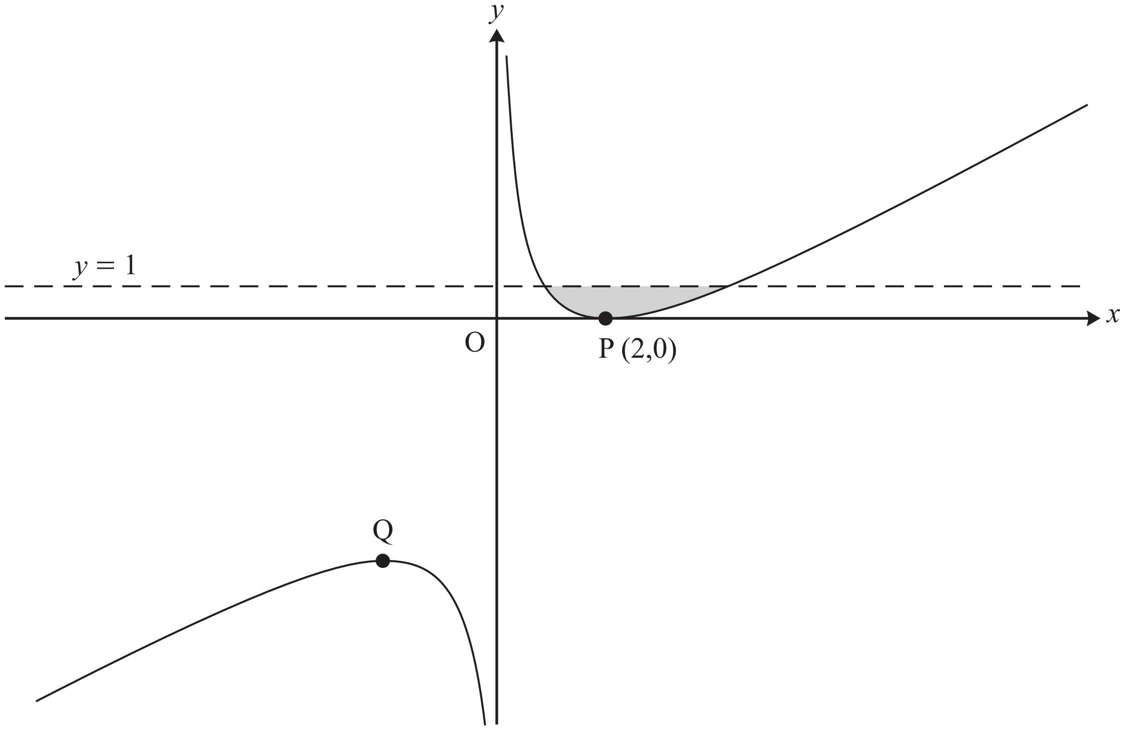

Fig. 8

(i) Show that $\mathrm{f}'(x) = 1 - \dfrac{4}{x^2}$, and find $\mathrm{f}''(x)$.

Hence find the coordinates of Q and, using $\mathrm{f}''(x)$, verify that it is a maximum point. **[7]**

(ii) Verify that the line $y = 1$ meets the curve $y = \mathrm{f}(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. **[6]**

The curve $y = \mathrm{f}(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = \mathrm{g}(x)$.

(iii) Show that $\mathrm{g}(x) = \dfrac{x^2 - 3x}{x+1}$. **[3]**

(iv) Without further calculation, write down the value of $\int_0^3 \mathrm{g}(x)\,\mathrm{d}x$, justifying your answer. **[2]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 2)^2 - 1, \quad x \in \mathbb{R}.$$

The curve crosses the x-axis at O and P, and has a turning point at Q.

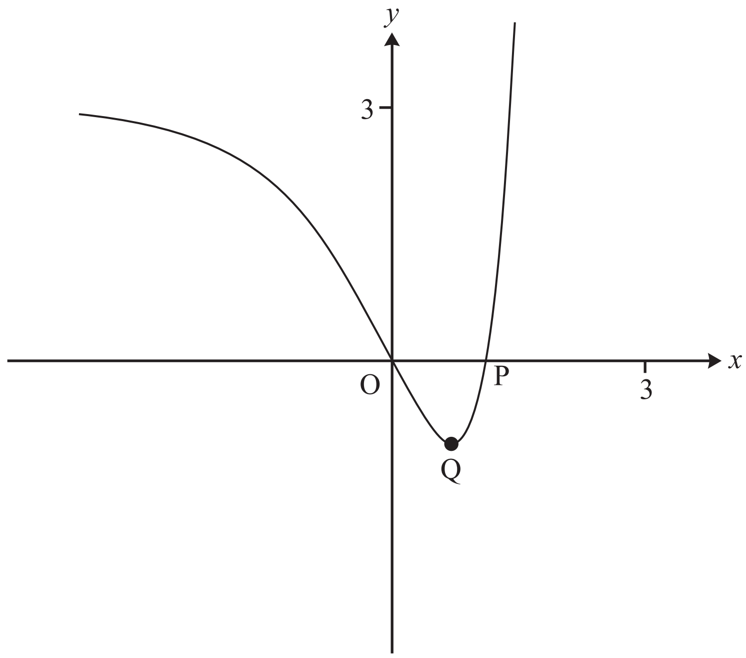

Fig. 9

(i) Find the exact x-coordinate of P. **[2]**

(ii) Show that the x-coordinate of Q is $\ln 2$ and find its y-coordinate. **[4]**

(iii) Find the exact area of the region enclosed by the curve and the x-axis. **[5]**

The domain of $\mathrm{f}(x)$ is now restricted to $x \geqslant \ln 2$.

(iv) Find the inverse function $\mathrm{f}^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. **[7]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1

2	

3	

4	

5

6 (i)	
6 (ii)	

7 (i)

7 (ii)

Turn over

Section B (36 marks)

8 (i)	

8 (ii)	

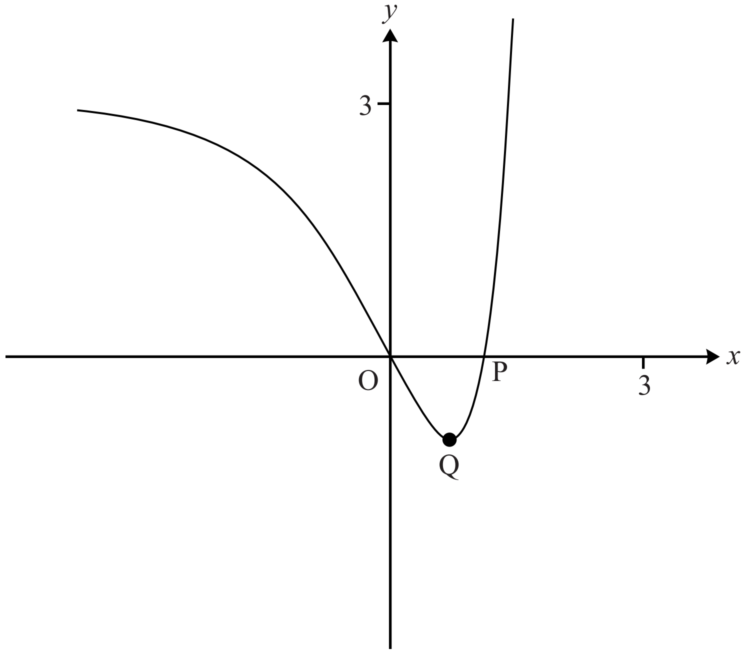
8 (iii)	
8 (iv)	

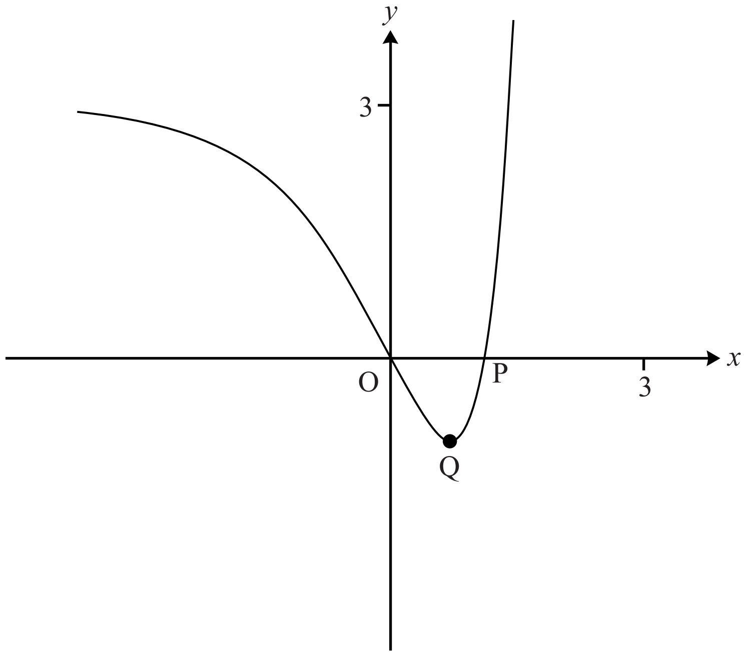
9 (i)	
9 (ii)	

9 (iii)	

9 (iv)

There is a spare copy of this graph on page 16.



9 (iv)	**Spare copy of graph for question 9(iv)** 

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$y = e^{2x}\cos x$	M1	product rule used	consistent with their derivs
			$\Rightarrow \quad dy/dx = 2e^{2x}\cos x - e^{2x}\sin x$	A1	cao – mark final ans	e.g. $2e^{2x} - e^{2x}\tan x$ is A0
			$dy/dx = 0 \Rightarrow e^{2x}(2\cos x - \sin x) = 0$	M1	their derivative = 0	
			$\Rightarrow \quad 2\cos x = \sin x$			
			$\Rightarrow \quad 2 = \sin x/\cos x = \tan x$	M1	$\sin x$ / $\cos x = \tan x$ used	or $\sin^2 x + \cos^2 x = 1$ used
			$\Rightarrow \quad x = 1.11$	A1	1.1 or 0.35π or better, or arctan 2, not 63.4° but condone ans given in both degrees and radians here	1.1071487 …, 0.352416…π, penalise incorrect rounding
			$\Rightarrow \quad y = 4.09$	A1cao	art 4.1	no choice
				[6]		
2			let $u = 2x - 1$, $du = 2\,dx$			
			$\int \sqrt[3]{2x-1}\,dx = \int \frac{1}{2}u^{\frac{1}{3}}\,du$	M1 M1	substituting $u = 2x - 1$ in integral × ½ o.e.	i.e. $u^{1/3}$ or $\sqrt[3]{u}$ seen in integral condone no du, or dx instead of du
			$= \frac{3}{8}u^{\frac{4}{3}} + c$	M1	integral of $u^{1/3} = u^{4/3}/(4/3)$ (oe) soi	not $x^{1/3}$
			$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have + c and single fraction mark final answer	so $\frac{3}{4}(2x-1)^{\frac{4}{3}} + c$ is M1M0M1A0
			or			
			$\int \sqrt[3]{2x-1}\,dx = ½ \times (2x - 1)^{4/3} \div 4/3$	M1 M1	$(2x - 1)^{4/3}$ seen ÷ 4/3 (oe) soi	e.g. correct power of $(2x - 1)$ e.g. ¾ $(2x - 1)^{4/3}$ seen
				M1	× ½	
			$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have + c and single fraction mark final ans	so $\frac{3}{8}(2x-1)^{\frac{4}{3}}$ is M1M1M1A0
				[4]		

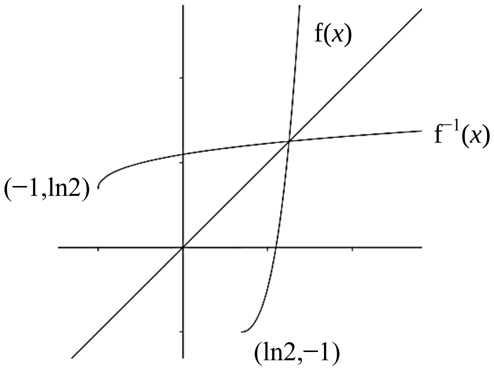
Question			Answer	Marks	Guidance	
3			let $u = \ln x$, $dv/dx = x^3$, $du/dx = 1/x$, $v = ¼ x^4$	M1	u, u', v', v all correct	
			$\int_1^2 x^3 \ln x \, dx = \left[\frac{1}{4}x^4 \ln x\right]_1^2 - \int_1^2 \frac{1}{4}x^4 . \frac{1}{x} dx$	A1	$\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 . \frac{1}{x} [dx]$	ignore limits
			$= \left[\frac{1}{4}x^4 \ln x\right]_1^2 - \int_1^2 \frac{1}{4}x^3 dx$	M1dep	simplifying $x^4 / x = x^3$ in second term (soi)	dep 1st M1
			$= \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4\right]_1^2$	A1cao	$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$ o.e.	
			$= 4\ln 2 - 15/16$	A1cao	o.e. must be exact, but can isw	must evaluate $\ln 1 = 0$ and combine $-1 + 1/16$
				[5]		
4			$h = r$ so $V = \pi h^3/3$	B1	o.e. e.g $\pi h^3 \tan 45° /3$	
			$dV/dt = 5$	B1	soi (can be implied from $V = 5t$)	e.g. from a correct chain rule
			$dV/dh = \pi h^2$	B1ft	must be dV/dh soi, ft their $\pi h^3/3$	but must have substituted for r
			$dV/dt = (dV/dh).\ dh/dt$	M1	any correct chain rule in V, h and t (soi)	e.g. $dh/dt = dh/dV \times dV/dt$,
			$\Rightarrow \quad 5 = 100\pi\ dh/dt$			
			$\Rightarrow \quad dh/dt = 5/100\pi = 0.016$ cm s^{-1}	A1	0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer	0.01591549… penalise incorrect rounding
			or $V = 5t$ so $\pi h^3/3 = 5t$	B1		
			$\Rightarrow \pi h^2 dh/dt = 5$	M1	or $5\ dt/dh = \pi h^2$ o.e.	
			$\Rightarrow dh/dt = 5/\pi h^2 = 5/100\pi = 0.016$ cm s^{-1}	A1	0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer	penalise incorrect rounding
				[5]		

Question			Answer	Marks	Guidance	
5			$y^2 + 2x \ln y = x^2$			
			$1^2 + 2\times1\times\ln1 = 1^2$ so (1, 1) lies on the curve.	B1	clear evidence of verification needed	at least "1 + 0 = 1"
				M1	d/dx (y^2) = 2ydy/dx	must be correct
			$2y\frac{dy}{dx} + 2\ln y + 2x.\frac{1}{y}.\frac{dy}{dx} = 2x$	M1	d/dx (2x ln y) = 2ln y + 2x/y dy/dx	must be correct
				A1 cao		condone dy/dx = … unless pursued
			[$\Rightarrow \frac{dy}{dx} = \frac{2x - 2\ln y}{2y + 2x/y}$]			
			when x = 1, y = 1, $\frac{dy}{dx} = \frac{2-2\ln1}{2+2}$	M1	substituting both x = 1 and y = 1 into their dy/dx or their equation in x, y and dy/dx	$2\frac{dy}{dx} + 2\ln1 + 2\frac{dy}{dx} = 2$
			= ½	A1cao	not from wrong working	
				[6]		
6	**(i)**		arcsin $x = \pi/6 \Rightarrow x = \sin \pi/6$	M1		
			= ½	A1	allow unsupported answers	
				[2]		
6	**(ii)**		$\sin \pi/4 = \cos \pi/4 = 1/\sqrt{2}$			
			$\Rightarrow$ arcsin $(1/\sqrt{2})$ = arccos $(1/\sqrt{2})$			
			$\Rightarrow$ $x = 1/\sqrt{2}$	B2	o.e. e.g. $\sqrt{2}/2$, must be exact; SCB1 0.707…	
				[2]		
7	**(i)**		$\text{ff}(x) = \text{f}(\frac{1-x}{1+x}) = \frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$	M1	substituting $(1-x)/(1+x)$ for x in f(x)	
			$= \frac{1+x-1+x}{1+x+1-x} = \frac{2x}{2} = x^*$	A1	correctly simplified to x **NB AG**	
			$\text{f}^{-1}(x) = \text{f}(x) = (1-x)/(1+x)$	B1	or just $\text{f}^{-1}(x) = \text{f}(x)$	

Question			Answer	Marks	Guidance	
				[3]		
7	**(ii)**		$g(-x)=\dfrac{1-(-x)^2}{1+(-x)^2}$	M1	substituting $-x$ for x in g(x) condone use of 'f' for g	if brackets are omitted or misplaced allow M1A0
			$=\dfrac{1-x^2}{1+x^2}=g(x)$	A1	must indicate that $g(-x) = g(x)$ somewhere	condone use of 'f' for g
			Graph is symmetrical about the y-axis.	B1	allow 'reflected', 'reflection' for symmetrical	must state axis (y-axis or $x = 0$)
				[3]		
8	**(i)**		$f'(x)=\dfrac{x.2(x-2)-(x-2)^2}{x^2}$	M1 A1	quotient (or product) rule, condone sign errors only correct exp, condone missing brackets here	e.g. $\dfrac{\pm x.2(x-2)\pm(x-2)^2}{x^2}$ PR: $(x - 2)^2.(-x^{-2}) + (1/x).2(x-2)$
			$=\dfrac{2x^2-4x-x^2+4x-4}{x^2}$			
			$= (x^2-4)/x^2 = 1-4/x^2$ *	A1	simplified correctly **NB AG**	with correct use of brackets
			or $f(x) = (x^2 - 4x + 4)/x$			
			$= x - 4 + 4/x$	M1 A1	expanding bracket and dividing each term by x correctly	must be 3 terms: $(x^2 - 4)/x$ is M0 e.g. $x - 4 + 2/x$ is M1A0
			$\Rightarrow f'(x) = 1 - 4/x^2$ *	A1	not from wrong working **NB AG**	
			$f''(x)=8/x^3$	B1	o.e. e.g. $8x^{-3}$ or $8x/x^4$	
			$f'(x) = 0$ when $x^2 = 4$, $x = \pm 2$	M1	$x = \pm 2$ found from $1 - 4/x^2 = 0$	allow for $x = -2$ unsupported
			so at Q, $x = -2$, $y = -8$.	A1	$(-2, -8)$	
			$f''(-2)$ [$= -1$] < 0 so maximum	B1dep **[7]**	dep first B1. Can omit -1, but if shown must be correct. Must state < 0 or negative.	must use 2nd derivative test

Question			Answer	Marks	Guidance	
8	(ii)		$f(1) = (-1)^2/1 = 1$ $f(4) = (2)^2/4 = 1$	B1	verifying f(1) = 1 and f(4) = 1	or $(x-2)^2 = x \Rightarrow x^2 - 5x + 4 = 0$ $\Rightarrow (x-1)(x-4) = 0,\ x = 1, 4$
			$\int_1^4 \frac{(x-2)^2}{x}\mathrm{d}x = \int_1^4 (x - 4 + 4/x)\mathrm{d}x$	M1	expanding bracket and dividing each term by x 3 terms: $x - 4/x$ is M0	if $u = x - 2$ $\int \frac{u^2}{u+2}\mathrm{d}u = \int (u - 2 + \frac{4}{u+2})\mathrm{d}u$
			$= \left[x^2/2 - 4x + 4\ln x\right]_1^4$	A1	$x^2/2 - 4x + 4\ln x$	$u^2/2 - 2u + 4\ln(u+2)$
			$= (8 - 16 + 4\ln 4) - (\frac{1}{2} - 4 + 4\ln 1)$			
			$= 4\ln 4 - 4\frac{1}{2}$	A1cao		
			Area enclosed = rectangle – curve	M1	soi	
			$= 3\times 1 - (4\ln 4 - 4\frac{1}{2}) = 7\frac{1}{2} - 4\ln 4$	A1cao	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
			or			
			Area = $\int_1^4 [1 - \frac{(x-2)^2}{x}]\mathrm{d}x$	M1	no need to have limits	
			$= \int_1^4 (5 - x - 4/x)\mathrm{d}x$	M1 A1	expanding bracket and dividing each term by x $5 - x - 4/x$	must be 3 terms in $(x-2)^2$ expansion
			$= \left[5x - x^2/2 - 4\ln x\right]_1^4$	A1	$5x - x^2/2 - 4\ln x$	
			$= 20 - 8 - 4\ln 4 - (5 - \frac{1}{2} - 4\ln 1)$ $= 7\frac{1}{2} - 4\ln 4$	A1cao **[6]**	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
8	(iii)		$[g(x) =]\ f(x+1) - 1$	M1	soi [may not be stated]	
			$= \frac{(x+1-2)^2}{x+1} - 1$	A1		
			$= \frac{x^2 - 2x + 1 - x - 1}{x+1} = \frac{x^2 - 3x}{x+1}$ *	A1 **[3]**	correctly simplified – not from wrong working **NB AG**	

Question			Answer	Marks	Guidance	
8	**(iv)**		Area is the same as that found in part (ii)	M1	award M1 for ± ans to 8(ii) (unless zero)	
			4ln4 – 7½	A1cao **[2]**	need not justify the change of sign	
9	**(i)**		At P, $(e^x - 2)^2 - 1 = 0$ $\Rightarrow \quad e^x - 2 = [\pm]1$, $e^x = [1 \text{ or}]\ 3$	M1	square rooting – condone no ±	
			or $(e^x)^2 - 4e^x + 3 = 0$ $\Rightarrow (e^x - 1)(e^x - 3) = 0$, $e^x = 1$ or 3	M1	expanding to correct quadratic and solve by factorising or using quadratic formula	condone e^x^2
			$\Rightarrow x = [0 \text{ or}]\ \ln 3$	A1 **[2]**	x-coordinate of P is ln 3; must be exact	condone P = ln 3, but not y = ln 3
9	**(ii)**		$f'(x) = 2(e^x - 2)e^x$ $= 0$ when $e^x = 2$, $x = \ln 2$ *	M1 A1 A1	chain rule correct derivative not from wrong working **NB AG**	e.g. 2 u × their deriv of e^x $2(e^x - 2)x$ is M0 or verified by substitution
			or $f(x) = e^{2x} - 4e^x + 3$ $\Rightarrow f'(x) = 2e^{2x} - 4e^x$ $= 0$ when $2e^{2x} = 4e^x$, $e^x = 2$, $x = \ln 2$ *	M1 A1 A1	expanding to 3 term quadratic with $(e^x)^2$ or e^{2x} correct derivative, not from wrong working or $2e^x(e^x - 2) = 0 \Rightarrow e^x = 2$, $x = \ln 2$ not from wrong working **NB AG**	condone e^x^2 or verified by substitution
			$y = f(\ln(2)) = -1$	B1 **[4]**		

Question			Answer	Marks	Guidance	
9	(iii)		$\int_0^{\ln 3}[(e^x-2)^2-1]dx=\int_0^{\ln 3}[(e^x)^2-4e^x+4-1]dx$	M1	expanding brackets must have 3 terms: $(e^x)^2 - 4$ is M0, condone e^x^2	or if $u = e^x$, $\int_1^3[u^2-4u+4-1]/u\,du$
			$=\int_0^{\ln 3}[e^{2x}-4e^x+3]dx$	A1	$\int e^{2x} - 4e^x + 3$ [dx] (condone no dx)	$= \int u - 4 + 3/u\ du$
			$=\left[\frac{1}{2}e^{2x}-4e^x+3x\right]_0^{\ln 3}$	B1 A1ft	$\int e^{2x} = ½\ e^{2x}$ [½ $e^{2x} - 4e^x + 3x$]	= [½ $u^2 - 4u + 3\ln u$]
			$= (4.5 – 12 + 3\ln 3) – (0.5 – 4)$ $= 3\ln 3 – 4$ [so area $= 4 – 3\ln 3$]	A1 **[5]**	condone $3\ln 3 – 4$ as final ans; mark final ans	
9	(iv)		$y = (e^x – 2)^2 – 1 \quad x \leftrightarrow y$ $x = (e^y – 2)^2 – 1$ $\Rightarrow \quad x + 1 = (e^y – 2)^2$	M1	attempt to solve for y (might be indicated by expanding and then taking lns)	or x if x and y not interchanged yet or adding (or subtracting) 1
			$\Rightarrow \quad \pm\sqrt{(x + 1)} = e^y – 2$ (+ for $y \geq \ln 2$)	A1	condone no ±	
			$\Rightarrow \quad 2 + \sqrt{(x + 1)} = e^y$ $\Rightarrow \quad y = \ln(2 + \sqrt{(x + 1)}) = f^{-1}(x)$	A1	must have interchanged x and y in final ans	
			Domain is $x \geq -1$ Range is $y \geq \ln 2$	B1 B1	must be $\geq$ and x (not y) or $f^{-1}(x) \geq \ln 2$, must be $\geq$ (not x or f(x)) if $x > -1$ and $y > \ln 2$ SCB1	if not specified, assume first ans is domain and second range
				M1 A1 **[7]**	recognisable attempt to reflect curve, or any part of curve, in $y = x$ good shape, cross on $y = x$ (if shown), correct domain and range indicated. [see extra sheet for examples]	$y = x$ shown indicative but not essential e.g. −1 and ln 2 marked on axes

Appendix 1 Annotation notes

1. All questions on practice and standardisation scripts should be fully annotated, unless they are all correct or worth no credit.
2. For all other scripts, each part question should be annotated according to the following guidelines:

   - if part marks awarded, annotate fully or:
   - If fully correct, one tick; if worth no credit, one cross, or a yellow line
   - if one mark only is lost, you can indicate this by A0 or B0 in the appropriate place
   - if 1 mark only is earned, you can indicate this by M1 or B1 in the appropriate place
   - if M0, then you need not annotate dependent A marks

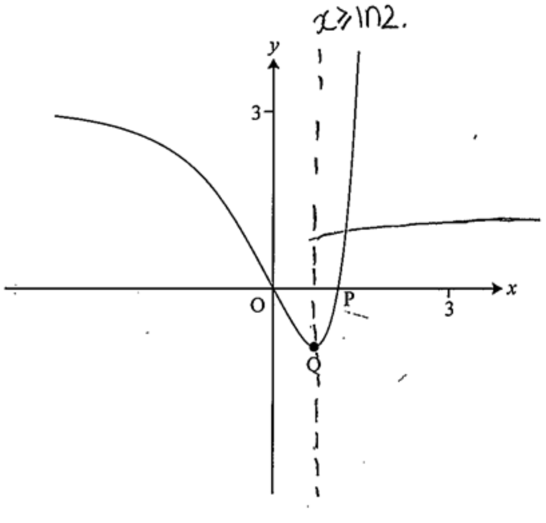
3. The following questions can be divided into sub-parts, which can be treated as above:

   - 8(i) First 3 marks, last 4 marks
   - 8(ii) First 4 marks, last 2 marks
   - 9(iv) first 3 marks, next 2 marks, last 2 marks

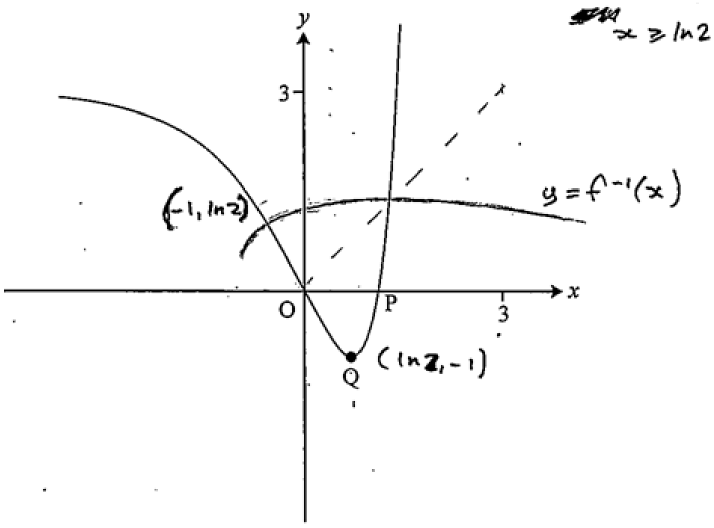
NB Please annotate all blank pages with a 'BP' mark (especially AOs), and blank answer spaces with a mark to show they have been seen (e.g. with a tick, a carat or yellow line) . Don't forget to scroll down to the bottom of each page, and annotate this if necessary to show this. Also, please indicate you have seen the spare copy of the graph in 9(iv)

Please put the annotations close to where they apply, rather than using the margins.

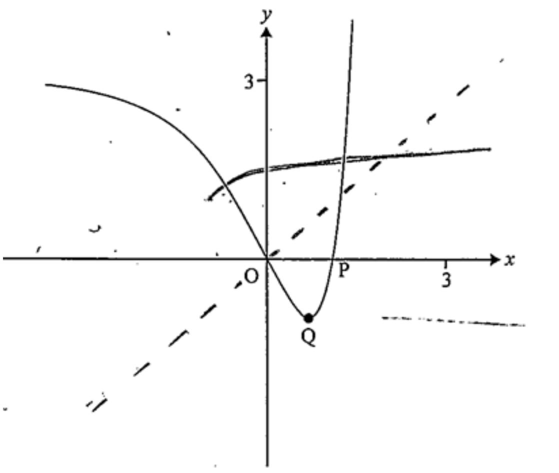
Appendix 2: Marked examples of 9(iv)



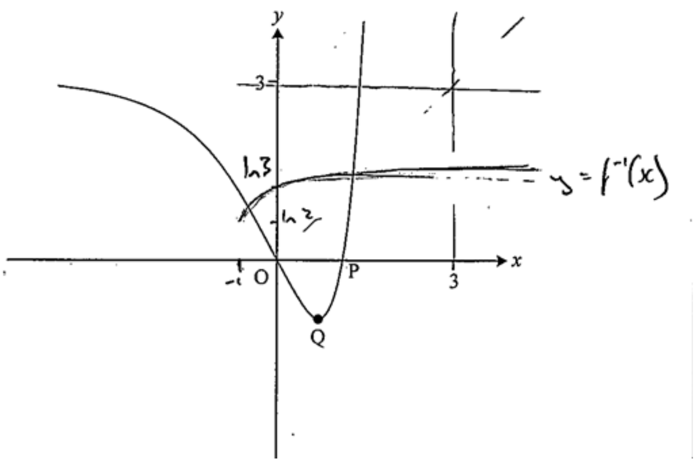
M1A0 (incorrect domain)



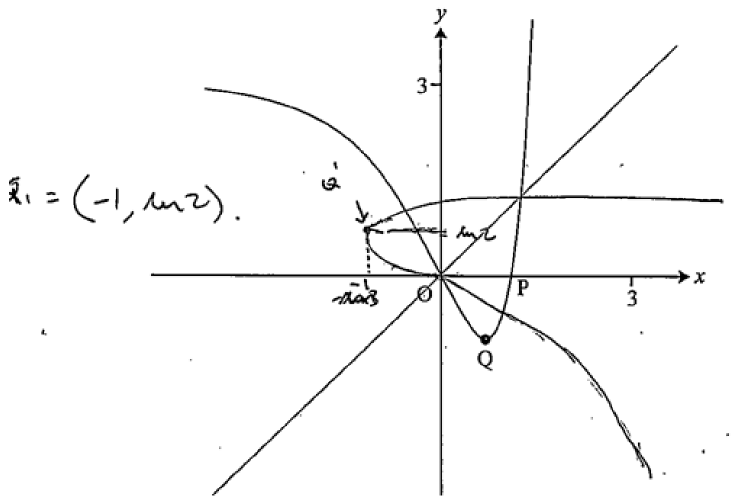
M1A1 (condone –ve gradient for x > 3)



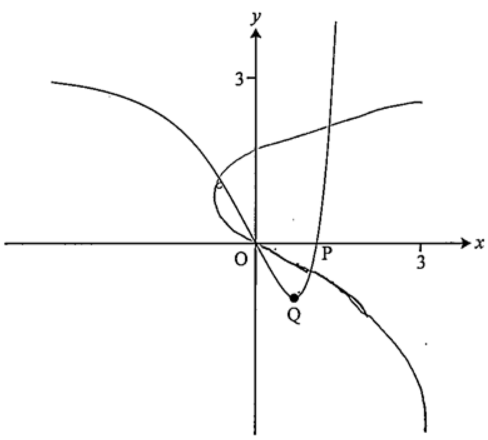
M1 A0 (domain not indicated)



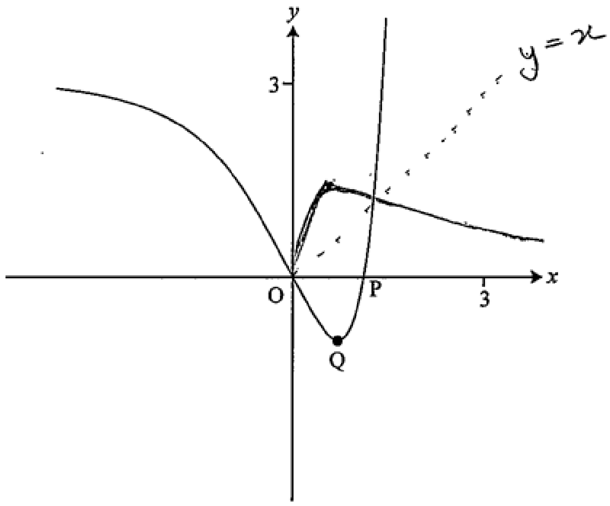
M1 A0



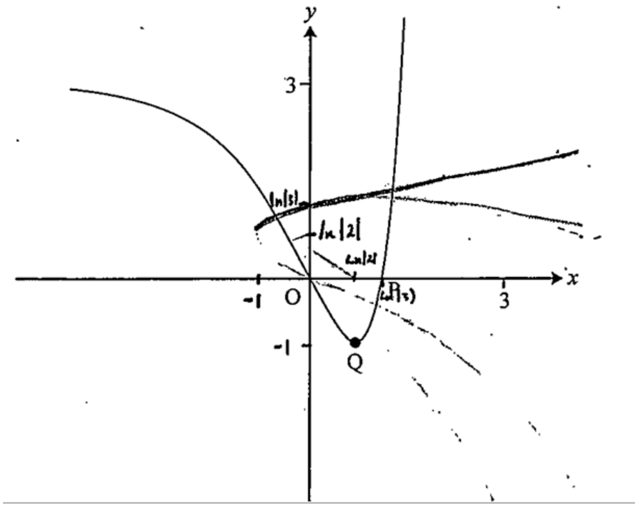
M1 A1



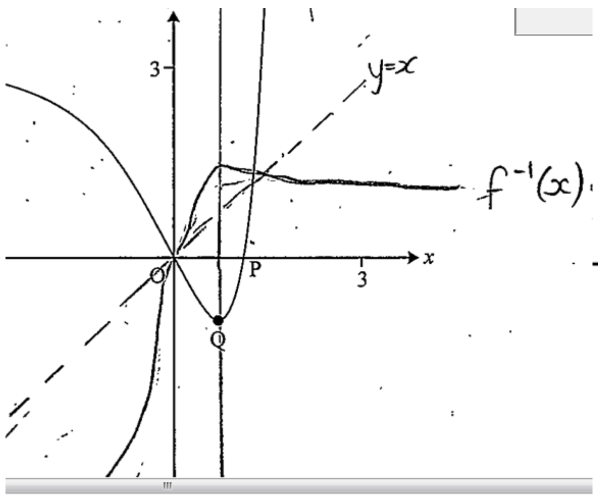
M1 A0



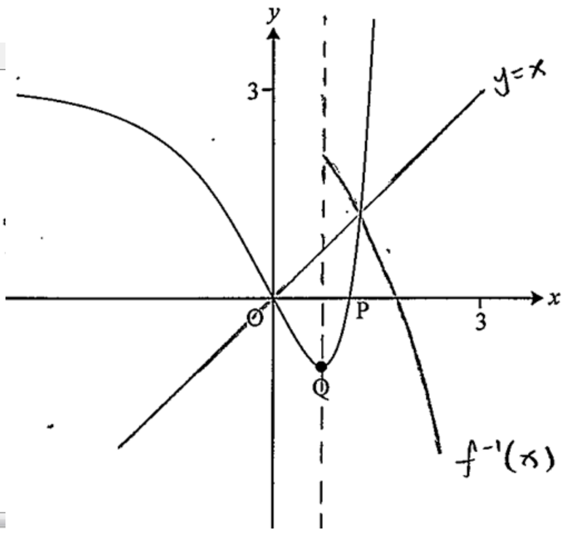
M1A0



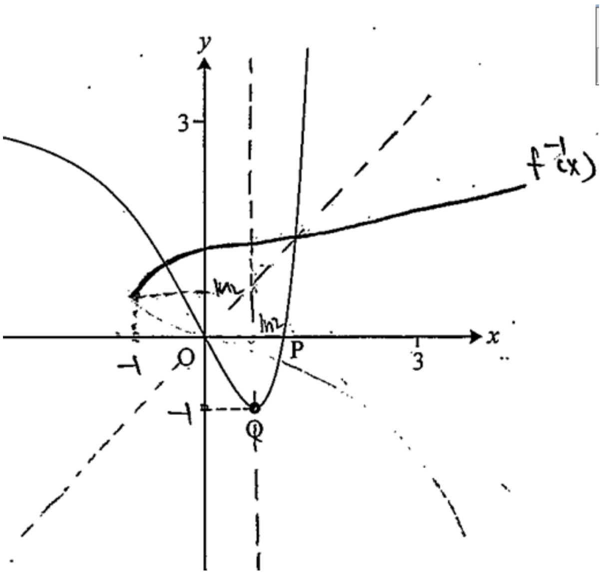
M1bodA1 (condone gradient at (-1, ln2)



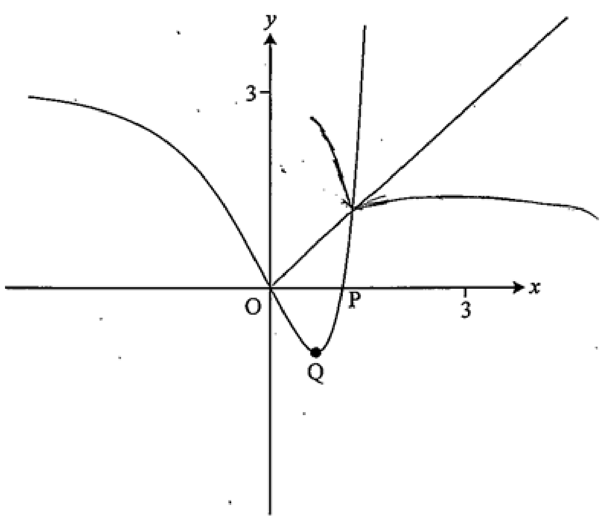
M1 A0



M1 A0



M1 A1



bod M1 A0 (evidence of reflection of part of curve here)

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4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments

The performance of candidates was similar and comparable to recent papers. There were many excellent scripts, showing sound preparation, good presentation, and accurate mathematics. The mean mark for the paper was very similar to last year's, though there were one or two quite demanding marks which meant there were fewer candidates who gained full marks. At the lower end, there were very few candidates who scored fewer than 25 marks. There was also very little evidence of candidates running out of time, and most answered all the questions.

More successful candidates have the experience to select appropriate methods. There is perhaps a danger that students are too eager to apply newly-taught methods, for example integration by substitution or parts, instead of more basic ones, such as expanding brackets and integrating term-by-term. This was evident in the number of students who lost marks for the integrations in 8(ii) and 9(iii). Conversely, they need to know when **not** to expand brackets, such as when finding the inverse function in 9(iv)!

Quite a few candidates used additional sheets – perhaps more space should have been allowed for some questions, such as 8(ii). However, it is helpful if centres avoid using 12-page additional booklets unless they are actually necessary, as all the pages have to be scanned and then checked and annotated.

Comments on Individual Questions

Section A

1. The product rule was done well, and most candidates were successful in arriving at tan x = 2 at the turning point. The most common error was to give x in degrees and then to use this to calculate y, giving a rather alarmingly large result!

2. This question was also answered well, either using substitution or by inspection. However, a surprising number of candidates who substituted left their final answer in terms of u, and a few lost the final mark through omitting the arbitrary constant.

3. There was a pleasing response to this question. Integration by parts was well understood by the majority of candidates, many of whom gained full marks. Very occasionally, u and v' were allocated to the wrong parts, and the other most common error was failing to simplify v u' before integrating this.

4. This question was less well done. Nearly all candidates gained marks for quoting a correct chain rule and using dV/dt = 5. By far the most common error thereafter was to fail to find V as a function of h and instead differentiating $V = \pi r^2 h/3$ to give $dV/dh = \pi r^2/3$. Even when candidates recognised the need to substitute for r, there were a surprising number of trigonometric errors, such as h = r sin 45°. A number of solutions which found dh/dt = 1/20π then went on to write or evaluate this as π/20.

5. Implicit differentiation was well understood, although differentiating the '2xlny' term using the product rule defeated some candidates, and there were some algebraic slips in re-arranging to find dy/dx (which virtually all candidates did before substituting x = 1 and y = 1).

6(i) This was an easy two marks for virtually all candidates, though occasionally they left their answer as sin π/6 without evaluating this as ½.

6(ii) This part was the polar opposite of part (i), with very few candidates getting anywhere. Two common errors were to infer that sin x = cos x and therefore $x = \pi/4$, and dividing arc sin x by arc cosx to get arc tan x. Successful candidates usually introduced another variable y equal to arc sin x, so that sin y = cos y, tan y = 1, $y = \pi/4$, and $x = \sin \pi/4 = 1/\sqrt{2}$.

7(i) Most candidates gained a method mark for substituting (1-x)/(1+x) for x in f(x). However, the simplification of the ensuing algebraic fraction proved to be problematic to many candidates, who failed to clear the subsidiary denominators correctly. Concluding that f-1(x) = f(x) should of course be a 'write down' from ff(x) = x; however, virtually all candidates found f-1(x) by rearranging the formula for x = f(y), usually correctly. Occasionally we were offered f-1(x) = 1/f(x) = (1+x)/(1-x).

7(ii) This was well answered, with few candidates using particular values of x to 'show' that g(x) was even. We condoned the use of f instead of g. Occasionally the brackets were misplaced in 1 – (-x)2 or 1 + (-x)2. The geometrical interpretation was well answered: although we would prefer 'symmetrical about the y-axis' to formulations such as 'reflection in the y-axis', the latter was nevertheless condoned.

Section B

8(i) This part was very well-answered, with many getting all 7 marks. The majority of candidates opted to use the quotient rule rather than the slightly easier method of expanding the numerator and dividing through by x. Even so, provided they took care in the use of brackets, they gained the first three marks. The second part was not quite as successful. Some candidates forgot to work out the y-coordinate of Q; others got the second derivative wrong, or failing to state explicitly that for a maximum the second derivative was negative.

8(ii) Most candidates verified that y = 1 when x = 1 or 4, though the majority did this by re-arranging the equation $(x - 2)^2 = x$ as a quadratic and solving this. However, for the integration, many candidates failed to spot the need to expand the $(x – 2)^2$ term and divide through by x, and most attempts to use substitution (except perhaps for the somewhat fatuous u = x) or parts usually led nowhere. Those who managed this integration successfully often failed to realise they then had to subtract this value from the area of the rectangle, or did this subtraction the wrong way round. Quite a few also calculated the area of the rectangle as 4 × 1 = 4 instead of 3.

8(iii) Fewer than half of the candidates gained full marks for this part of the question. Many did not know how the translation would affect the function algebraically, for example starting with y - 1 = f(x + 1). Of those who derived a correct expression for g(x), many failed to incorporate the '−1' into their fraction.

8(iv) Very few candidates scored both marks here. A method mark was awarded if their answer indicated recognising the translation of their area from part (ii); but few of those who got part (ii) correct realised that the integral would be the negative of this as the transformed area is below the axis.

9(i) Most candidates succeeded in finding x = ln 3, either by square rooting or solving the quadratic in e^x. The second method was somewhat compromised by setting $x = e^x$ (rather than a different variable) to get a quadratic in x, though we condoned this for both marks.

9(ii) This provided a simple four marks for most candidates, using a chain rule to find the derivative, setting this to zero and solving to get x = ln 2. A neat alternative method was to recognise that the $(e^x – 2)^2$ term must be non-negative and minimum when $e^x – 2 = 0$, or x = ln 2.

9(iii) This proved to be a rather costly part for candidates unless they recognised the requirement to multiply out $(e^x – 2)^2$ -1 to get $e^{2x} – 4e^x + 3$ and then integrate term-by-term. Other attempts using substitution or parts usually got nowhere. Although originally we required candidates to give the area as 4 – 3ln3, very few actually did this, so it was decided to condone a (negative) area of 3ln3 – 4.

9(iv) Rather more than half of the candidates managed the inverse function well, though a few made errors at the last stage of taking the square root, and concluded with $y = \ln(\sqrt{(x + 1)})+ 2$, or $y = \ln(\sqrt{(x + 1)} + \ln 2$. Some were perhaps encouraged by the previous part to multiply out $(e^x – 2)^2$ again, though they could still obtain a method mark for a step towards finding y in terms of x. It was not uncommon to see candidates taking logs of individual terms.

The domain and range required accurate use of notation (x = … for domain, y = … or $f^{-1}(x)$ = … for range, and ≥ for both); occasionally these were correct but in the wrong order, so that it was unclear which was which.

The graph proved to be quite demanding. The majority of candidates convinced us they were attempting to reflect some of the original curve in y = x and got a method mark. However, far fewer gained the ‘A’ mark by reflecting the correct part of the graph from (-1, ln 2) onwards, and identifying this point clearly in their sketch.

GCE Mathematics (MEI)				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0